Benha University
Faculty Of Engineering at Shoubra

ECE 122
Electrical Circuits (2)(2017/2018)
Lecture (02)
Parallel Resonance

Prepared By :
Dr. Moataz Elsherbini
motaz.ali@feng.bu.edu.eg

Remember

Series Resonance

[1] $\omega_0 = \frac{1}{\sqrt{LC}}$, $f_0 = \frac{1}{2\pi\sqrt{LC}}$

[2] $Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$ → total series Res.

$Z = R$ (at resonance).

[3] $BW = \omega_2 - \omega_1 = \frac{R}{L} = \frac{\omega_0}{Q}$

[4] $\omega_{1\,actual} = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} \cong \omega_0 - B/2$ → from half Power

$\omega_{2\,actual} = \frac{+R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} \cong \omega_0 + B/2$ → from half Power

} at $Z = \sqrt{2}R$

[5] $\omega_0 = \sqrt{\omega_1 \omega_2}$

[6] $Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$

Series Resonance Circuit (Cont.)

EXAMPLE 20.5 A series *R-L-C* circuit is designed to resonant at $\omega_s = 10^5$ rad/s, have a bandwidth of $0.15 f_s$, and draw 16 W from a 120-V source at resonance.

a. Determine the value of *R*.
b. Find the bandwidth in hertz.
c. Find the nameplate values of *L* and *C*.
d. Determine the Q_s of the circuit.

$$BW = f_2 - f_1 = \frac{R}{2\pi L}$$

a. $P = \frac{E^2}{R}$ and $R = \frac{E^2}{P} = \frac{(120\text{ V})^2}{16\text{ W}} = \mathbf{900\ \Omega}$

b. $BW = 0.15 f_s$ $\quad f_s = \frac{\omega_s}{2\pi} = \frac{10^5\text{ rad/s}}{2\pi} = 15{,}915.49\text{ Hz}$

$$BW = 0.15 f_s = 0.15(15{,}915.49\text{ Hz}) = \mathbf{2387.32\ Hz}$$

c. $BW = \frac{R}{2\pi L}$ and $L = \frac{R}{2\pi BW} = \frac{900\ \Omega}{2\pi(2387.32\text{ Hz})} = \mathbf{60\ mH}$

$f_s = \frac{1}{2\pi\sqrt{LC}}$ and $C = \frac{1}{4\pi^2 f_s^2 L} = \frac{1}{4\pi^2(15{,}915.49\text{ Hz})^2(60 \times 10^{-3}\text{ H})}$
$= \mathbf{1.67\ nF}$

d. $Q_s = \frac{X_L}{R} = \frac{2\pi f_s L}{R} = \frac{2\pi(15{,}915.49\text{ Hz})(60\text{ mH})}{900\ \Omega} = \mathbf{6.67}$

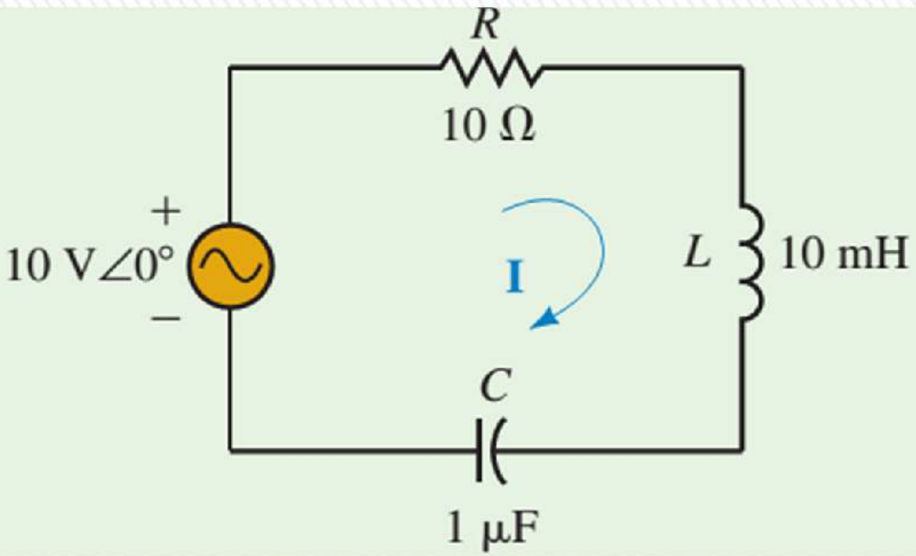
Ex 2

a. Determine the maximum power dissipated by the circuit.

b. Use the results obtained from Example 21–1 to determine the bandwidth of the resonant circuit and to arrive at the approximate half-power frequencies, ω_1 and ω_2.

c. Calculate the actual half-power frequencies, ω_1 and ω_2, from the given component values. Show two decimal places of precision.

d. Solve for the circuit current, I, and power dissipated at the lower half-power frequency, ω_1, found in part (c).



Solution

a. $$P_{max} = \frac{E^2}{R} = 10.0 \text{ W}$$

b. From Example 21–1, we had the following circuit characteristics:

$$Q_S = 10,\ \omega_S = 10 \text{ krad/s}$$

The bandwidth of the circuit is determined to be

$$\text{BW} = \omega_S Q_S = 1.0 \text{ krad/s}$$

If the resonant frequency were centered in the bandwidth, then the half-power frequencies occur at approximately

$$\omega_1 = 9.50 \text{ krad/s}$$

and

$$\omega_2 = 10.50 \text{ krad/s}$$

c. $\omega_1 = -\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$

$$= -\frac{10\ \Omega}{(2)(10\ \text{mH})} + \sqrt{\frac{(10\ \Omega)^2}{(4)(10\ \text{mH})^2} + \frac{1}{(10\ \text{mH})(1\ \mu\text{F})}}$$

$$= -500 + 10\,012.49 = 9512.49\ \text{rad/s} \quad (f_1 = 1514.0\ \text{Hz})$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

$$= 500 + 10\,012.49 = 10\,512.49\ \text{rad/s} \quad (f_2 = 1673.1\ \text{Hz})$$

d. At $\omega_1 = 9.51249$ krad/s, the reactances are as follows:

$$X_L = \omega L = (9.51249\ \text{krad/s})(10\ \text{mH}) = 95.12\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(9.51249\ \text{krad/s})(1\ \mu\text{F})} = 105.12\ \Omega$$

The current is now determined to be

$$\mathbf{I} = \frac{10\ \text{V}\angle 0°}{10\ \Omega + j95.12\ \Omega - j105.12\ \Omega}$$

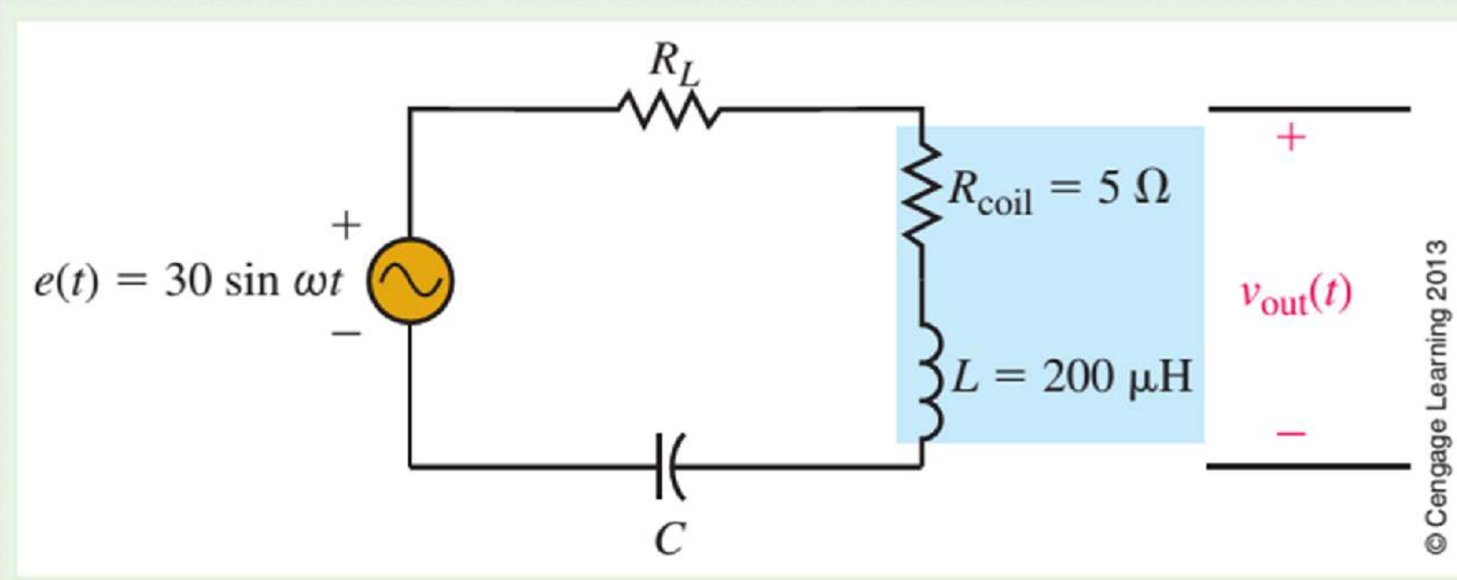
$$= \frac{10\ \text{V}\angle 0°}{14.14\ \Omega\angle -45°}$$

$$= 0.707\ \text{A}\angle 45°$$

and the power is given as

$$P = I^2R = (0.707\ \text{A})^2(10\ \text{V}) = 5.0\ \text{W}$$

Ex 3



a. Calculate the values of R_L and C for the circuit to have a resonant frequency of 200 kHz and a bandwidth of 16 kHz.

b. Use the designed component values to determine the power dissipated by the circuit at resonance.

c. Solve for $v_{out}(t)$ at resonance.

a. Because the circuit is at resonance, we must have the following conditio

$$Q_S = \frac{f_S}{\text{BW}}$$

$$= \frac{200 \text{ kHz}}{16 \text{ kHz}}$$

$$= 12.5$$

$$X_L = 2\pi f L$$

$$= 2\pi(200 \text{ kHz})(200\ \mu\text{H})$$

$$= 251.3\ \Omega$$

$$R = R_L + R_{\text{coil}} = \frac{X_L}{Q_s}$$

$$= 20.1\ \Omega$$

and so R_L must be

$$R_L = 20.1\ \Omega - 5\ \Omega = 15.1\ \Omega$$

Since $X_C = X_L$, we determine the capacitance as

$$C = \frac{1}{2\pi f X_C}$$

$$= \frac{1}{2\pi(200 \text{ kHz})(251.3 \ \Omega)}$$

$$= 3.17 \text{ nF } (\equiv 0.00317 \ \mu\text{F})$$

b. The power at resonance is found from Equation 21–20 as

$$P_{\text{max}} = \frac{E^2}{R} = \frac{\left(\frac{30 \text{ V}}{\sqrt{2}}\right)^2}{20.1 \ \Omega}$$

$$= 22.4 \text{ W}$$

$$\mathbf{V}_{\text{out}} = \frac{(R_1 + j\omega L)}{R}\mathbf{E}$$

$$= \frac{(5\ \Omega + j251.3\ \Omega)}{20.1\ \Omega} 21.21\ \text{V}\angle 0^\circ$$

$$= (251.4\ \Omega\angle 88.86^\circ)(1.056\ \text{A}\angle 0^\circ)$$

$$= 265.5\ \text{V}\angle 88.86^\circ$$

which in time domain is given as

$$v_{\text{out}}(t) = 375 \sin(\omega t + 88.86^\circ)$$

Ex 4

It is required to broadcast a <u>shoubra radio</u> station to be detected through your FM radio. Design a suitable series RLC circuit to verify this mission. The station must be heard within bandwidth of 2MHz, while the most purity sound heard at 90MHz.

Sol/ $\because$ $BW = \frac{R}{L}$ (rad/sec)

$\therefore (2 \times 10^6 \times 2\pi) = \frac{R}{L}$

let $\boxed{R = 100\,\Omega}$ مثلاً $\therefore$ $L = \frac{100}{2 \times 10^6 \times 2\pi}$

يفضل وضع R صغيرة في حالة series $\cong 7.95\,\mu H$

$\because \omega = \frac{1}{\sqrt{LC}}$ $\therefore 2\pi \times 90 \times 10^6 = \frac{1}{\sqrt{7.95 \times 10^{-6} \times C}}$

$\therefore \boxed{C \cong 0.39\,pF}$

Ex 5

Refer to the circuit of Figure 2.

a. Determine the resonant frequency expressed as ω (rad/s) and f(Hz).

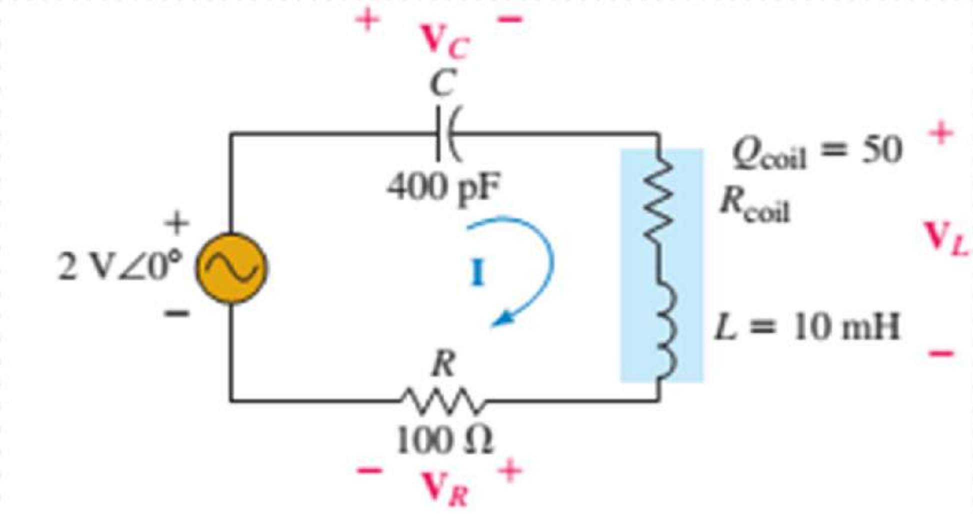
b. Calculate the total impedance, Z_T, at resonance.

c. Solve for current I at resonance.

d. Solve for V_R, V_L, and V_C at resonance.

e. Calculate the power dissipated by the circuit and evaluate the reactive powers, Q_C and Q_L.

f. Find the quality factor, Q_S, of the circuit.



Sol

a- $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(400\times10^{-12})(10\times10^{-3})}} = 500\, krad/s$

$f = \frac{\omega}{2\pi} = 79.617\, kHz$

b- $Z_t|_{res} = R + R_{coil} = 100 + R_{coil}$

$\therefore Q_{coil} = \frac{\omega L}{R_{coil}} \therefore R_{coil} = \frac{\omega L}{Q} = \frac{500\times10^{3} \times 10\times10^{-3}}{50} = 100$

$\therefore Z_t = R = 100 + 100 = 200\angle 0$

c- $I_{res} = \frac{V}{Z_{tres}} = \frac{V}{R_t} = \frac{2}{200} = 0.01A\angle 0 = 10mA\angle 0$

d- $V_R = IR\angle 0 = (0.01)(100) = 1\angle 0$

$V_C = IX_C = (0.01)(\frac{1}{\omega C})\angle -90 = 0.01 \frac{1}{(500)(10^3)(400)(10^{-12})}\angle -90 = 50\angle -90$

$V_L = I(X_L + R) = (0.01)(R + jX_L)$

$= (0.01)(100 + j(500\times10^{3})(10\times10^{-3})) = (0.01)(100 + j5000)$

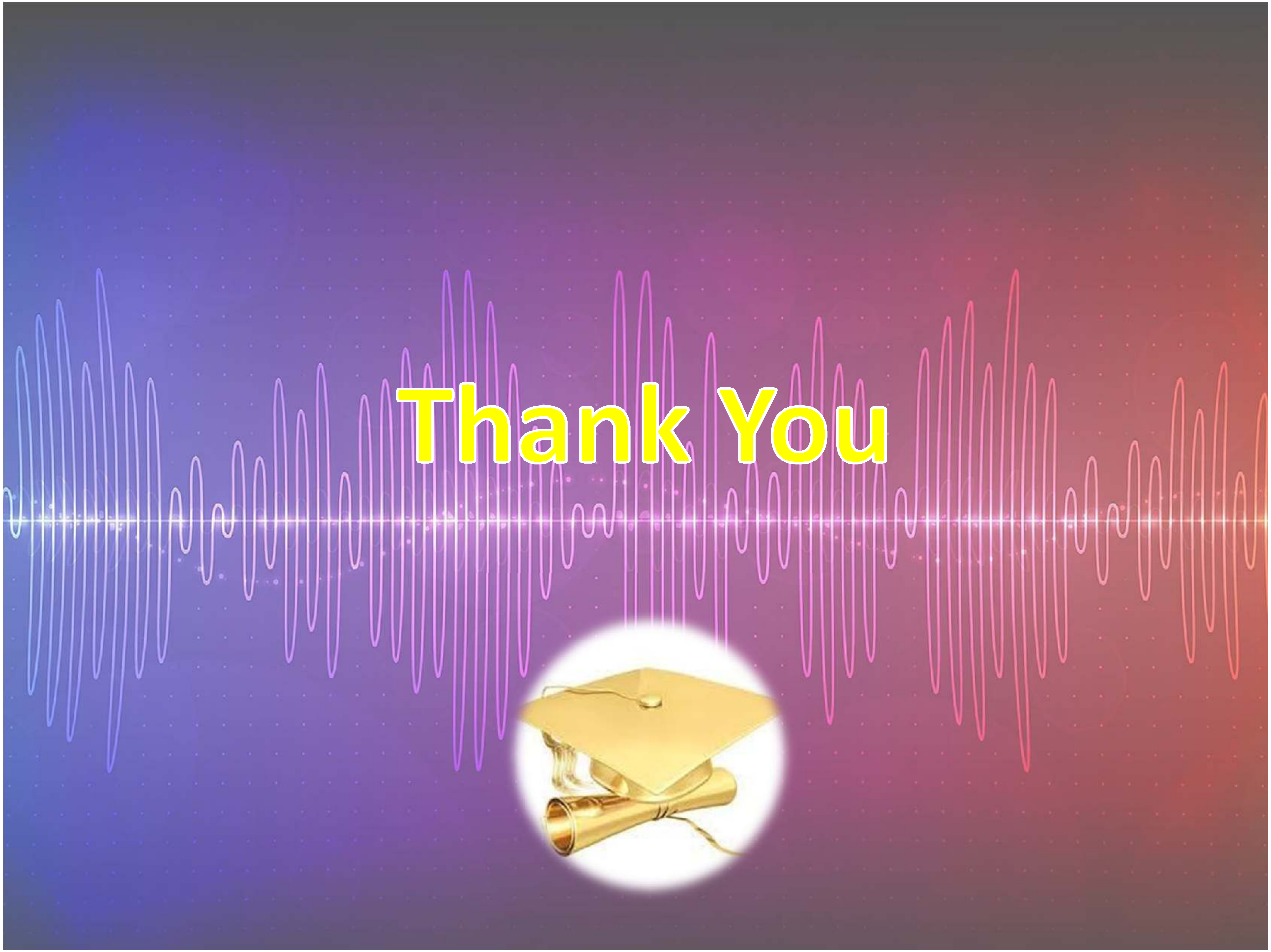
$= 1 + j50 = \sqrt{1^2 + 50^2}\angle \tan^{-1}(\frac{50}{1}) = 50.01\angle 88.85^\circ$

e- $P = VI = (2)(0.01) = 0.02 = 20mW$

$Q_C = I^2 X_C = (0.01)(50)\angle -90 = 0.5\ VAR$

$Q_L = I^2 X_L = (0.01)(50.01) = 0.501\ VAR$

$Q = \frac{\omega L}{R_t} = \frac{(500\times10^{3})(10\times10^{-3})}{200} = 25$ *


Thank You